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Large extra dimensions from a small extra dimension

Zackaria Chacko, Patrick J. Fox, Ann E. Nelson and Neal Weiner

Department of Physics, Box 1560, University of Washington
Seattle, WA 98195-1560, USA
E-mail: zchacko@phys.washington.edu, pjfox@phys.washington.edu, anelson@phys.washington.edu, nealw@phys.washington.edu

Abstract: Models with extra dimensions have changed our understanding of the hierarchy problem. In general, these models explain the weakness of gravity by diluting gravity in a large bulk volume, or by localizing the graviton away from the standard model. In this paper, we show that the warped geometries necessary for the latter scenario can naturally induce the large volumes necessary for the former. We present a model in which a large volume is stabilized without supersymmetry. We comment on the phenomenology of this scenario and generalizations to additional dimensions.

Keywords: Extra Large Dimensions, Classical Theories of Gravity.

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Contents

1. Introduction

One of the most exciting developments in the past few years has been the recognition that theories with extra dimensions can radically change our understanding of old problems. That extra dimensions could be relevant for four dimensional theories is not a new idea, going back to the theories of Kaluza and Klein [1, 2, 3]. However, much of the recent interest has been sparked by the possibilities that extra dimensions offer to change our understanding of the hierarchy problem, which, simply put, is the question of the origin and stability of the large ratio of the Planck scale M_{Pl} to the electroweak symmetry breaking scale M_W. For a recent review and more references, see ref. [4].

In ref. [5, 6], Arkani-Hamed, Dimopoulos and Dvali ("ADD") and Antoniadis noted that with n compact extra dimensions, and factorizable geometry with volume V_n, the effective four dimensional Planck scale M_{Pl} was related to the higher-dimensional gravitational scale M_* by the relation

$$M_{Pl}^2 = M_*^{2+n} V_n \, . \tag{1.1}$$

They then proceeded to consider the possibility that V_n was exponentially large, such that $M_* \sim M_W$. Remarkably, for $n \geq 2$, such a scenario was phenomenologically viable, provided the standard model fields are confined to a 3+1 dimensional subspace — a "3-brane". The hierarchy problem is recast not as a question of why M_W is small compared to M_{Pl}, but rather, why V_n is so large.

In ref. [7], ("RS1"), Randall and Sundrum considered an alternative possibility. Rather than focus on large, factorizable compact dimensions, they noted that a non-factorizable "warped" product of a fifth dimension with our four had extremely interesting implications. Specifically, the scale factors of Poincaré invariant 3-branes embedded at different locations in 5D anti-de Sitter space (AdS) differ exponentially. Hence obtaining an exponential hierarchy of scales merely required a slice of 5D AdS between two 3-branes. Gravity was localized to the brane with a large warp factor ("the Planck brane"), while standard model particles were localized to the brane with the small warp factor ("the TeV brane"). To obtain a sufficiently large hierarchy, the size of the fifth dimension had to be about 40 in units of the fundamental scale. Goldberger and Wise demonstrated that such a dimension could naturally be stabilized with a bulk scalar field [8], providing a solution to the hierarchy problem. In a second paper [9], ("RS2"), Randall and Sundrum showed that such localization of gravity could obviate the need for compactification of the fifth dimension.

Models have been proposed to generate the exponentially large compact volumes of the ADD scenario [10, 11]. However these contain massless bulk scalars. Also, the bulk cosmological constant must be extremely small. (This requirement is in addition to the fine-tuning needed to make the effective 4-D cosmological constant sufficiently small.) Thus explaining the hierarchy via large extra dimensions seemed to require fine-tuning of several parameters. Bulk supersymmetry can render these fine-tunings natural [12, 13], but ideally one would not need to invoke supersymmetry to solve the hierarchy problem with large extra dimensions. Also, these models contain light radions, with mass less than an meV, which pose a severe challenge for cosmology [14, 15]. Given the dearth of models, it has been difficult to examine the validity of various "model independent" claims about the required features of the stabilization mechanism, and the radion phenomenology and cosmology.

In this paper, we will present a mechanism capable of generating exponentially large compact dimensions without any supersymmetry. We take advantage of the fact that in warped six (and higher) dimensional geometry, the Randall-Sundrum mechanism can be used to generate a hierarchy of scales. However, rather than using the mechanism to localize gravity on another brane, we will use it to generate an an exponentially large volume for one or more compact dimensions. We will show that such exponentially large volumes can be stabilized with a scalar field, analogous to the Goldberger-Wise mechanism. We give a specific example, which is very similar to the six-dimensional Randall-Sundrum type model considered by Chacko and Nelson [16]. In that model gravity was localized to a 4-brane ("the Planck brane") with a large warp factor, while the standard model resided on a different brane with a small warp factor ("the TeV brane"). One dimension of the 4-brane was compact, but had a large size in terms of the fundamental scale. The main new feature of the example considered here is that we now place the standard model on the same 4-brane which localizes gravity, and there is no need for a second 4-brane. The standard model particles are confined to a 3-brane which is embedded in the 4-brane. Gravity is weak due to the presence of a large compact dimension, and the fact that gravity spreads evenly over the entire 4-brane.

The central elements of our scenario are simple and can be generalized to any number of compact dimensions.

1. We will require a $3+n$ brane wrapped around n compact dimensions of a warped $5+n$ dimensional space-time. These n dimensions play the role of the large extra dimensions of the ADD scenario, and we will refer to them as "ADD dimensions". The warp factors of the ADD dimensions as well as of our usual 4 depend on an additional dimension, which we refer to as the RS dimension.

2. Due to the presence of a non-trivial warp factor, exponentially large hierarchies can arise naturally. The physics which stabilizes the size of the ADD dimensions is sensitive to these warpings. As a consequence, the volume of the ADD space can be quite large.

3. The particles of the standard model are confined to a three brane, which is embedded in the $3{+}n$ brane. Gravity, of course, lives everywhere, but is mostly localized to the $3{+}n$ brane. Gravity appears weak to us because it is diluted by the large ADD dimensions.

The layout of the paper is as follows: in the next section 2 we consider a toy six-dimensional model with one 4-brane. The requirement that the metric be regular everywhere stabilizes the ADD dimension, while adding a bulk scalar field will naturally achieve a large volume.

In section 3, we discuss the possibilities for generalizations to higher dimensions. In section 4 we discuss the phenomenology of our scenario.

2. A simple model

To achieve a sufficiently large volume such that the fundamental scale, M_*, is as low as $\sim$ TeV, we must realistically generate a large volume for at least two additional dimensions. For simplicity, here we present a example in which an exponentially large volume is generated for one additional dimension. Generalizations to more dimensions should follow straightforwardly and we comment on them in section 3.

The setup will be simple: we consider the case of a six dimensional space with two extra compact directions. We will wrap a 4-brane around one of these dimensions and take the space to be orbifolded across it. We take the bulk cosmological constant to be negative (AdS). Without bulk matter, the space will want to expand to infinity. To stabilize the compact dimensions, we will add a massive bulk scalar field with a source on the brane. If the mass of this scalar is lighter than the fundamental scale by a factor of a few, the setup will be stabilized at finite but exponentially large volume.

We label a general coordinate by x^M where M takes values from 0 to 3,5 and 6. The four dimensional coordinates are labeled by x^μ while the two extra coordinates are r and ϕ. The coordinate ϕ runs from 0 to 2π. The space is assumed to be orbifolded about the 4-brane which is at a specific location $r = b$ in the higher dimensional space. In addition to this 4-brane, the sources of gravity are a bulk cosmological constant Λ_B, and a bulk scalar field which has a source on the brane. We also assume another form of matter which is localized to the brane and whose effect is to make the brane tension anisotropic, such as a flux [17, 12], the Casimir energy of massless fields [16] or a complex scalar field with a non-trivial winding number in the compact direction.

The gravitational action for our system is given by

$$S_G = \int d^6x\sqrt{-G}\left[(2M_*^{D-2}R - \Lambda_B) - \sqrt{-\bar{G}}\delta(r-b))\bar{\Lambda}\right]. \tag{2.1}$$

The action for the bulk scalar field is given by

$$S_M = \int d^6x \sqrt{-G} \frac{1}{2}(-\partial_M \psi \partial^M \psi - m^2 \psi^2) + \sqrt{-\bar{G}} H(\psi) \delta(r-b) \, . \tag{2.2}$$

The action for the fields localized to the brane takes the form

$$S_W = \int d^6x \sqrt{-\bar{G}} \delta(r-b) L_W \, . \tag{2.3}$$

We will obtain an approximate solution for the metric of this coupled gravity-matter system using the method of ref. [16]. We first obtain a metric solution for the gravitational part of the action alone. We then solve for the scalar field in this background as a perturbation. The backreaction of the scalar field on the metric then determines the solution of this system to leading order in the source for the scalar field.

For simplicity we will be taking

$$H(\psi) = \lambda \psi \, . \tag{2.4}$$

The metric for this system has the general form

$$ds^2 = f(r) \eta_{\mu\nu} dx^\mu dx^\nu + s(r) d\phi^2 + dr^2 \, . \tag{2.5}$$

The solution for the metric from the gravitational part of the action alone for $r < b$ is given by [18] (see also [16, 19, 20]),

$$\begin{aligned} f_0(r) &= \frac{\cosh^{4/5} \alpha r}{\cosh^{45} \alpha b} \, , \\ s_0(r) &= \frac{\sinh^2 \alpha r}{\alpha^2 \cosh^{6/5} \alpha r} \, . \end{aligned} \tag{2.6}$$

Here we are normalizing f to be one at the location of the brane and α is defined by[1]

$$\alpha^2 = -\frac{5}{16} \frac{\Lambda_B}{2M_*^4} \, . \tag{2.7}$$

The corresponding solutions for $r > b$ are determined by the symmetry condition of the orbifold.

We will see shortly that this setup is not an extremum at finite b. Hence, we will delay our discussion of the matching conditions until we have included the additional scalar field and asymmetric brane tensions. The corrections to this geometry due to the scalar field are parametrized by

$$\begin{aligned} f &= f_0(1+\epsilon) \\ s &= s_0(1+\kappa) . \end{aligned} \tag{2.8}$$

[1] This definition of α differs from that in [16].

The Einstein equations linearized in ϵ and κ take the form

$$\begin{aligned}
\frac{3}{2}\epsilon'' + 3\frac{f_0'}{f_0}\epsilon' + \frac{1}{2}\kappa'' + \frac{s_0'}{2s_0}\kappa' + \frac{3}{4}\frac{f_0'}{f_0}\kappa' + \frac{3}{4}\frac{s_0'}{s_0}\epsilon' &= \overline{T}\,, \\
2\epsilon'' + 5\frac{f_0'}{f_0}\epsilon' &= \overline{T}\,, \\
3\frac{f_0'}{f_0}\epsilon' + \frac{s_0'}{s_0}\epsilon' + \frac{f_0'}{f_0}\kappa' &= \tilde{T}\,,
\end{aligned} \tag{2.9}$$

where $\overline{T}$ and $\tilde{T}$ are related to the stress tensor T for the bulk scalar by

$$\overline{T} = \frac{T^0{}_0}{2M_*^4} = \frac{T^5{}_5}{2M_*^4}, \tag{2.10}$$

$$\tilde{T} = \frac{T^6{}_6}{2M_*^4}. \tag{2.11}$$

They are not independent but are constrained by energy-momentum conservation $\nabla_M T^{MN} = 0$ which when linearized implies

$$\tilde{T}' = \frac{(f^2 s^{1/2})'}{f^2 s^{1/2}}(\overline{T} - \tilde{T})\,. \tag{2.12}$$

The linearized 55 equation above can be solved to give

$$\epsilon' = f^{-5/2}\int_0^r d\rho \frac{1}{2} f^{5/2}\overline{T}\,. \tag{2.13}$$

Substituting for $\overline{T}$ in terms of $\tilde{T}$ in this expression and integrating by parts this reduces to

$$\epsilon' = \frac{\tilde{T}}{4\alpha}\tanh(2\alpha r) + \frac{1}{4}\mathrm{sech}^2(\alpha r)\int_0^r d\rho \tanh^2(2\alpha\rho)\tilde{T}\,. \tag{2.14}$$

For large αr keeping only the leading and subleading terms in an expansion in $e^{-2\alpha r}$ this further reduces to

$$\epsilon' = \frac{1}{4\alpha}\tilde{T} + D(r)e^{-2\alpha r}\,, \tag{2.15}$$

where

$$D(r) = \int_0^r d\rho \tanh^2(2\alpha\rho)\,\tilde{T}\,. \tag{2.16}$$

Using the linearized 66 Einstein equation, we solve for κ' in the same region $\alpha r \gg 1$

$$\kappa' = \frac{1}{4\alpha}\tilde{T} - 4D(r)e^{-2\alpha r}\,. \tag{2.17}$$

We can now address the stability of the setup by investigating the matching conditions at the brane. Linearizing in ϵ and κ, these take the form

$$\frac{3}{2}\Delta\frac{f_0'}{f_0} + \frac{1}{2}\Delta\frac{s_0'}{s_0} + \frac{3}{2}\Delta\epsilon' + \frac{1}{2}\Delta\kappa' = \frac{1}{2}\lambda\psi(b) - \beta^2\,, \tag{2.18}$$

$$2\Delta\frac{f_0'}{f_0} + 2\Delta\epsilon' = \frac{1}{2}\lambda\psi(b) - \gamma^2\,, \tag{2.19}$$

where $\beta^2 = T^{*0}_{\ 0}/2M_*^4$ and $\gamma^2 = T^{*5}_{\ 5}/2M_*^4$. Here T^* is the brane tension. The anisotropy between β^2 and γ^2 is due to the matter field localized on the wall, which we are keeping at lowest order in our calculation. As discussed in ref. [16], for large αb the anisotropy is an exponentially small effect.

Taking the difference of equations (2.18) and (2.19), and using the expressions obtained earlier for ϵ' and κ', we can find an equation for $D(b)$. Using the explicit forms for f_0 and s_0, we have

$$-5D(b)e^{-2\alpha b} = \beta^2 - \gamma^2 - 4\alpha\,\mathrm{csch}(2\alpha b)\,. \tag{2.20}$$

Notice that without the scalar field, $D(b)$ vanishes. $\beta^2 - \gamma^2 \sim 1/M_*^4 s^{-5/2}$ which for large b is $O\left(\alpha e^{-2\alpha b}\right)$. Thus if $(\beta^2 - \gamma^2)/e^{-2\alpha b} > 8\alpha$, then, without the scalar field, (2.20) can only be satisfied (trivially) with $b = \infty$. The scalar field provides an extra attractive force which stabilizes the setup. With the scalar field present and the inequality satisfied there is a solution for finite b.

It is also interesting to note that $D(b)$, which is a small, subleading effect, is critical in determining b. This suggests that there will be a light scalar mode in the lower dimensional effective theory, analogous to the radion of the Goldberger-Wise model [21].

To determine b, we will obtain an approximate expression for $D(b)$. To do this we consider the equation of motion for the bulk scalar:

$$-\psi'' - \left(2\frac{f_0'}{f_0} + \frac{1}{2}\frac{s_0'}{s_0}\right)\psi' + m^2\psi = \lambda\delta(r-b)\,, \tag{2.21}$$

which, given the forms of f_0 and s_0 is just

$$-\psi'' - 2\alpha\coth(2\alpha r)\psi' + m^2\psi = \lambda\delta(r-b)\,. \tag{2.22}$$

While this equation is difficult to solve exactly, it simplifies greatly in in large and small r limits. For our purposes, it will be sufficient to obtain approximate solutions, using the asymptotic forms

$$\psi = A_1 I_0(mr) \approx A_1\left(1 + \frac{1}{4}m^2r^2 + \cdots\right) \qquad r \ll \frac{1}{\alpha}\,, \tag{2.23}$$

$$\psi = B_1 e^{\sigma_1 r} + B_2 e^{\sigma_2 r} \qquad \frac{1}{\alpha} \ll r < b\,. \tag{2.24}$$

Here σ_1 and σ_2 are given by

$$\begin{aligned}\sigma_1 &= -\alpha - \sqrt{\alpha^2 + m^2}\,,\\ \sigma_2 &= -\alpha + \sqrt{\alpha^2 + m^2}\,.\end{aligned} \tag{2.25}$$

To obtain an approximate expression for D we now assume that the form of (2.23), valid for $r \ll 1/\alpha$, holds out to a point $a \approx 1/\alpha$, and that the form of (2.24) holds from the point a all the way to b. This will suffice for an order of magnitude estimate of b. Further we will be interested in the case $m^2 \ll \alpha^2$. Then $\sigma_1 \approx -2\alpha$ and $\sigma_2 \approx m^2/(2\alpha)$. Matching values and first derivatives at $r = a = 1/\alpha$, and satisfying the jump condition at $r = b$, we find

$$A_1 \approx \frac{\alpha\lambda}{m^2}e^{-bm^2/2\alpha}\,, \qquad B_1 \approx \frac{(1-a\alpha)e^{2a\alpha}\lambda}{4\alpha}e^{-bm^2/2\alpha}\,, \qquad B_2 \approx \frac{\alpha\lambda}{m^2}e^{-bm^2/2\alpha}\,. \tag{2.26}$$

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The expression for the stress tensor is

$$\tilde{T} = \frac{1}{8M_*^4}(\psi'^2 - m^2\psi^2)\,. \tag{2.27}$$

In order that the stress tensor of the scalar field is a perturbation on the background metric we assume that $\lambda \ll \alpha^3$. With these approximations, we find

$$D(b) = \frac{\alpha^3\lambda^2}{8M_*^4 m^4}\left(e^{-bm^2/\alpha} - 1\right). \tag{2.28}$$

At this order in m^2, this result comes entirely from that large r region in the integral of eq. (2.16), justifying the approximations of equations (2.23), (2.24). The equation determining $D(b)$, eq. (2.20), then gives

$$e^{-bm^2/\alpha} = 1 + \frac{8M_*^4 m^4}{5\alpha^3\lambda^2}(8\alpha - Y_C)\,, \tag{2.29}$$

where Y_C is a constant of $O(M_*)$ parametrizing the asymmetry in the 4-brane tension. $Y_C = (\beta^2 - \gamma^2)/e^{-2\alpha b}$, for large b. For a solution to exist at finite b with the scalar field present the anisotropy on the brane must be large enough, i.e. $Y_C > 8\alpha$.

In order to first obtain a quick estimate we assume that M_* and m^2/λ are both of order one in appropriate powers of α. This yields [16]

$$b \approx O\left(\frac{\alpha}{m^2}\right). \tag{2.30}$$

In this limit $\alpha b \gg 1$ since $\alpha^2 \gg m^2$, and hence the volume of the extra dimension is large.

More generally, assuming the right-hand side of eq. (2.29) to be some number $1/Q < 1$, we have

$$e^{bm^2/\alpha} \approx Q \Rightarrow e^{b\alpha} \approx Q^{\alpha^2/m^2}\,. \tag{2.31}$$

The radius of the large dimension is then roughly

$$r_c = V \approx \alpha^{-1} e^{2b\alpha/5} = \alpha^{-1} Q^{2\alpha^2/5m^2} = \alpha^{-1} e^{\left(\frac{2\alpha^2}{5m^2}\ln Q\right)}\,. \tag{2.32}$$

For α/m and Q of order a few, this dimension is exponentially large, realizing the scenario of [5] for one dimension.

3. Modifications and generalizations

A number of alternatives are possible to stabilize the ADD dimensions. Instead of using the regularity of the metric at the origin, we could have instead had an "inner brane" at some location $r = a$, where s and f are exponentially small. The brane spacing could be stabilized using a bulk scalar field. The inner brane could lie at an orbifold fixed point of a compact dimension, or, we could extend the space beyond the inner brane to a regular origin, as we have done here. In the former case, dynamics on the inner brane would determine the volume of the ADD space, which would naturally stabilize the size to the inverse of the (exponentially small) scale of inner brane physics.

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Energy	ADD	RS1	RS2	heterotic
$E < 1/r_c$	SM + 4D GR +radions	SM + 4D GR + radions	SM + 4D GR + power law corr.	SM + 4D GR + radions
$E \sim 1/r_c$	SM + 4D GR + grav. KK modes	see below ($1/r_c \sim$ TeV)	"	SM + 4D GR + grav. KK modes +radion
$1/r_c < E \lesssim$ TeV	SM + (4+n)D GR	NA	"	SM + (4+n)D GR +power law corr. + radion
$E \sim$ TeV	strong quant. grav.	SM+ 4D GR + grav. KK modes	"	strong quant. grav.
$E \gtrsim$ TeV	"	5D AdS quant. grav. + 4D GR (+ other 5D fields)	"	"
$E \sim M_{Pl}$	"?	strong quant. gravity	strong quant. gravity	"?

Table 1: Summary of effective theories at various scales in the ADD, RS and heterotic extra dimensional scenarios.

It should be straightforward to generalize this mechanism to more dimensions in order to make it phenomenologically viable with a TeV quantum gravity scale. The most straightforward generalization would be to compactify ADD dimensions on hyperspheres, in which case regularity at the origin would still be well defined, and could be used to stabilize their size. Alternatively, one could employ concentric $n-$tori, and use the dynamics of the interior brane to determine the overall volume of the setup.

4. Phenomenology

The phenomenology of our scenario is very similar to that of the ADD proposal [10], however the presence of the RS dimension leads to a some interesting distinctions. The relation between the Planck scale and M_* is approximately

$$M_{Pl}^2 = \left(\frac{M_*^3}{\alpha}\right) M_*^n V_n \,, \tag{4.1}$$

which is quite similar to (1.1), but with an additional factor of M_*/α from the RS dimension. Note that (1.1) would give the same result for n large dimensions and one smaller dimension of size $1/\alpha$. However an important difference is that the gravitational modes of the RS dimension cannot be neglected in the effective theory below the scale $1/\alpha$, as those whose wave functions are small in the large warp factor region are very light, with masses of order $1/r_c$. These light modes are very weakly coupled on the brane, but their effects will show up as nearly power law corrections to the n dimensional gravitational potential, as in the RS2 model. In principle the modes of both the RS and ADD dimensions could

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show up in collider searches for higher dimensional graviton emission at high energies. The radion for the RS dimension will be somewhat lighter than the scale $1/r_c$ and also may produce observable deviations from $1/r^2$ gravitational forces at long distances, although its wavefunction on the brane is small.

Because our scenario shares interesting features of both the ADD and RS extra dimensional models, we refer to it as "heterotic".[2]

In table 1 we summarize and compare the main features of the phenomenology of the heterotic scenario with those of ADD, RS1, and RS2, by describing the relevant effective theories as a function of scale. We use the abbreviations "SM" for Standard Model, "nD GR" for n-dimensional General Relativity, and "KK" for the (Kaluza-Klein) higher dimensional modes of bulk fields.

5. Summary

In this paper, we have offered a mechanism to stabilize exponentially large dimensions, using the ideas of Randall-Sundrum and Goldberger-Wise for generating and stabilizing exponentially different scales in warped geometry. It is amusing that warped geometry, the central element in the RS1 and RS2 scenarios [7, 9], in combination with additional compact dimensions, can naturally yield a scenario very similar to ADD.

A great deal remains to be studied, including generalizations to higher dimensions, models with interior branes, the effects of our 3-brane on the geometry, the mass(es) and coupling(s) of the radion(s), and cosmology.

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